



Modeling of Seawater Intrusion Applied in the Coastal Aquifer of the Quaternary Sands of the Niayes Area, Mboro-Senegal

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Abstract Groundwater resources in the coastal aquifer of the Niayes region are overexploited. Therefore, dissolved salts are the main contaminants in the groundwater of the area combined with those due to the invasion of salt water, caused mainly by economic activities related to industry and irrigation as well as domestic activities. One of the most significant effects of spatially varying pumping from the aquifer of north littoral is the intensification of seawater intrusion. This paper deals the methods of prevention of seawater encroachment, predicting the location and movement of the saltwater interface in the areas closed to Mboro. The focus of this work is to develop two major approaches: the first one approach is the development of an analytic model and the second is the development of a finite volume and finite difference simulation model using different management scenarios. An analytic model based on sharp interface approach is calibrated and validated by using hydraulic head observations. A numerical model based on transition zone approach is verified using the calibrated model data. Initial heads and concentration were simulated using a MARTHE computational code and simulation results are compared with previously analytic solutions where good agreement is observed.

Keywords Seawater intrusion, Niayes area, sharp interface, modeling, contaminants, Senegal

1. Introduction

In the Niayes area, saltwater intrusion can have serious consequences in terms of environmental and economic impacts. The Niayes area is an agricultural region, which spreads from Dakar to Saint-Louis with a length of 180 km by 20 km wide, brings together nearly 80% of the vegetable and horticultural production in Senegal [1]. Groundwater resources in coastal around the Niayes area are overexploited [2, 3]. One of the most important impacts of over-pumping in a coastal aquifer is the intensification of seawater intrusion.

Hence, there is a need to predict the location and movement of the potential contamination fronts. Practical management requires knowledge of not only the present response, but also of the long-term transient response. For these managerial purposes, a numerical model can assist in estimating the location of the freshwater/saltwater interface for given sets of hydrological conditions.

The saltwater intrusion problem is modeled using an approach giving rise to a transition zone between two fluids [4]. The combined effect of these mechanisms gives rise to a transition zone with variable solute concentration. Simulation of the transition zone separating freshwater and saltwater requires simultaneous solution of the governing equations of fluid flow and solute transport. Such an approach leads to density-dependent transport models that are limited in their field application by computational constraints [5]. A second approach to modeling saltwater intrusion is the sharp interface approach. In this case, the saltwater and



freshwater are considered as two immiscible fluids separated by an interface. The main approximation is provided from the Ghyben-Herzberg [5,6] works that were latter generalized in several models [7]. This approximation need to be carefully verified before application.

Thus, the main objective of this work is to determine the position of the freshwater/saltwater interface in the coastal aquifer of the Niayes area using two modeling methods. First a Mathematical modeling using an analytical solution [8, 9]. For this analytical solution, the freshwater and saltwater flow equations are processed simultaneously to determine the position of the interface. Then, we proceeded by numerical modeling from the MARTHE computational code [10]. The code allowed us to simulate and predict solute concentrations in the coastal aquifer niayes.

2. Materials and Methods

2.1. Study Area

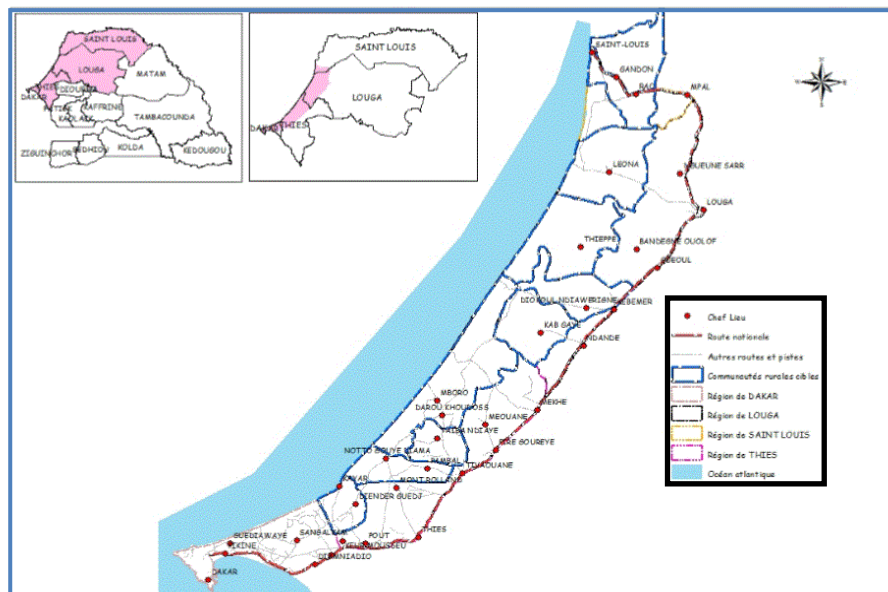


Figure 1: Location of the study area

The North littoral of Senegal (denominated Niayes) area registers administratively in the four regions bordering the maritime fringe of northern Senegal including Dakar, Thiès, Louga and Saint-Louis. It extends over a length of about 180 km and on a 20 to 30 km wide depending on the studies [1]. It is located between latitude $14^{\circ}3$ and $16^{\circ}N$, then longitudes 16° and $17^{\circ}5$ W [1, 11]. Figure 1 shows the area with the main municipalities concerned. With decentralization, several municipalities have been built on this coastal strip. They are Sangalcam, Diender, Bayakh, Kayar, Notto Gouye Diamo, Mboro, Darou Khoudos, Gandon, Léona and Thieppe, all of them agricultural.

The "Niayes" are closed inter-dune depressions with a flush or uplifted aquifer and are characterized by a succession of dunes and depressions. The Niayes zone includes several ecological units and contains very important biological and mining resources whose exploitation deserves special attention.

In the hydrogeological context, the Quaternary sand aquifer contained in the tank of the marine and continental [8] mainly occupy the Niayes area. The groundwater recharge is performed by rainwater infiltration and its productivity often reaches $100 \text{ m}^3/\text{h}$ [11].

2.2. Mathematical Model

Modelling involves a large number of issues, ranging from understanding site geology to analytical or numerical methods and the way to use these in order to represent the studied aquifer. This work starts by describing the sharp interface approach and its analytical solution. But there are many questions about its validity. This section concludes with a discussion on methods by several authors. Gionnoulopoulos in 2004 [12] studied the



application validity of the sharp interface approach and the optimal amount of water that can be pumped. The study is based on the assumption that the sharp interface approach can be applied if the transition zone is narrow relative to the domain dimension [11]. Pool and Carrera [13] have introduced a corrective factor to properly associate the Ghyben-Herzberg relationship with the critical pumping rate of coastal aquifers.

a. The modeling procedure

We consider the coordinates $(x, z) \in \mathbb{R}^N$ ($N=1$ or 2 for the aquifers) and the time $t \in [0, +\infty]$ [where x is the horizontal coordinate, z the vertical one and t the time].

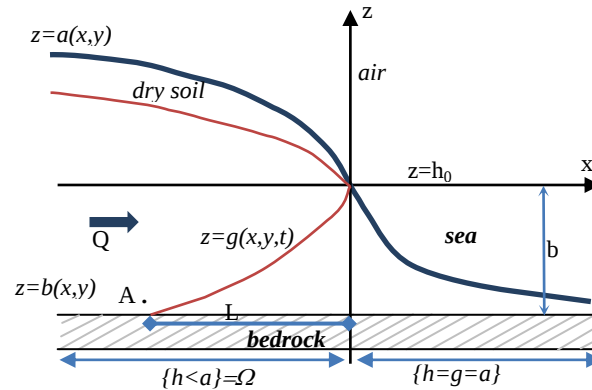


Figure 2: Physical model of seawater intrusion in a coastal aquifer

We note $\forall x \in \mathbb{R}^N, b(x) \leq a(x)$. In addition, the interface separating fresh water and the soil surface is described by $\Gamma_h^t = \{(x, z) \in \mathbb{R}^N, z=h(x, t)\}$ and the separating fresh and saline groundwater is given by $\Gamma_g^t = \{(x, z) \in \mathbb{R}^N, z=g(x, t)\}$. Physically, fluid density of saline water is bigger than fresh water, which allows us to write $g(x) \leq h(x)$ for all $(x, t) \in \mathbb{R}^N \times [0, +\infty]$. Finally, we note that all the functions defined previously will satisfy the following condition: $b(x) \leq g(x, t) \leq h(x, t) \leq a(x)$ for all $(x, t) \in \mathbb{R}^N \times [0, +\infty]$. The aquifer system is divided in two sets: $\Omega_d(t) = \{(x, z) \in \mathbb{R}^N, g(x, t) < z < h(x, t)\}$, the open set of fresh water; $\Omega_s(t) = \{(x, z) \in \mathbb{R}^N, b(x) < z < g(x, t)\}$, the open set of saline water.

b. Governing equations of flow in a phreatic aquifer

Freshwater-seawater model for groundwater flow with free surface consists of two vertically integrated governing equations, one for freshwater flow and the other for saltwater flow [6, 12] :

$$\begin{cases} \phi \frac{\partial(h-g)}{\partial t} = \text{div}(-(h-g)K\nabla h) + q \\ \phi_s \frac{\partial(g-b)}{\partial t} = \text{div}(-(g-b)K_s\nabla h_s) + q_s \end{cases} \quad (1)$$

Where ϕ is the porosity (%), h is hydraulic head (m), g is position of the interface between fresh water and saline water (m), K hydraulic conductivity (m/s) and q is fluid mass source (m^3/jour).

2.3. Presentation of the analytical solution

Albitar [14] has proposed this analytical solution. To solve the seawater intrusion model, the following assumptions are necessary.

Sharp interface between fresh and saline water; the phreatic aquifer of the Niayes area is homogeneous; the direction of flow is horizontal; the current lines follow the salty front and cannot cross it except near the sea surface. This solution shows the idealized Ghyben-Herzberg model of an interface in a coastal phreatic aquifer. Essentially, Ghyben and Herzberg assume static equilibrium and a hydrostatic pressure distribution in the fresh water region, with stationary seawater. The equipotentials are vertical line or surfaces (identical to the Dupuit assumption).

a. Resolution of governing equations

The Darcy's Law is expressed by:

$$Q = -T\nabla h \quad (2)$$



Where T is transmissivity (m²/s), h is hydraulic head (m) and Q is rate of flow (m³/s). The Transmissivity is given by:

$$T = KH = K(h + g) \quad (3)$$

Where H (m) is the thickness of the aquifer saturated by water, g (m) position of the fresh/saline water interface, K (m/s) the hydraulic conductivity. The law of conservation of the flow of water by infiltration is given by:

$$Q - (Q + \frac{dQ}{dx} \Delta x) + I \Delta x = 0 \quad (4)$$

The recharge was determined from the water cycle balance equation:

$$I = P - (E + ETR + \Delta S) \quad (5)$$

Where P the quantity of measured rain (mm), E is the stream of water (mm), ETR is real evapotranspiration (mm) and ΔS is the stock in the soil (mm), I is infiltration or recharge (mm/an).

Because the flow is steady and assumed horizontal, we have used the Ghyben-Herzberg relationship

$$g = \delta \times h \quad (6)$$

Where $\delta = \frac{1}{\varepsilon}$; with $\varepsilon = \frac{\rho_s - \rho}{\rho}$ is density difference ratio, ρ (kg/m³) is the density of fresh water and ρ_s (kg/m³) is density of saline water.

Now we obtain after putting equation 4 in equation 2 (Darcy's Law)

$$-\frac{K}{2\delta^2} (1 + \delta) (2g \frac{dg}{dx}) = Q \quad (7)$$

Because of the steady state, the velocity of water is constant and the following equation

$$\left. \frac{dg^2}{dx} \right|_{x=0} = -\frac{2Q_0\delta^2}{K(1+\delta)} \quad (8)$$

Or equation 4 can be rewrite as below:

$$\frac{dg^2}{dx} = -\frac{2I\delta^2}{K(1+\delta)} x + c_1 \quad (9)$$

From equation (7) we can determine c_1 and equation (8) becomes:

$$g^2 = \frac{I\delta^2}{K(1+\delta)} x^2 - \frac{2Q_0\delta^2}{K(1+\delta)} x + c_2 \quad (10)$$

The position of the interface is given by:

$$g^2 = h_0^2 - \frac{\delta^2}{K(1+\delta)} [2Q_0 + Ix]x \quad (11)$$

If there is no recharge, the interface is a portion of parabolic equation:

$$g^2 = h_0^2 - \frac{2Q_0\delta^2}{K(1+\delta)} x \quad (12)$$

Where h_0 is the thickness (m).

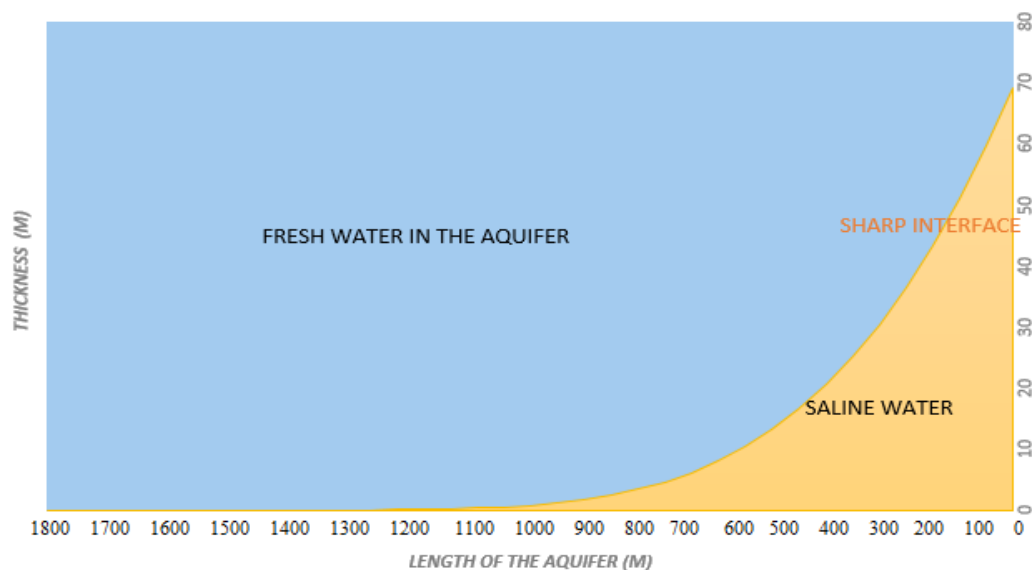
b. Results and discussions of the analytical model solution

To realize these simulations the input values parameters are given in the table n°1. We realize the series of simulations in a homogeneous rectangle with 1800 m and 80 m without well. The substratum is considered as the level of reference $b = 0$ m. We have programmed the solutions by using Matlab software and the results have been exploited in Excel programming.



Table 1: Parameters of simulations in a general case

Parameters	Values
Rate of flow pumping, Q	1,29 m ³ /h
Hydraulic conductivity, K	4,356.10 ⁻³ m/s
Aquifer length, L_x	1800 m
Average recharge estimated, I	8,7.10 ⁻⁸ m/s
Aquifer thickness, h_0	80 m
Density of fresh water, ρ	1000 kg/m
Density of saline water, ρ_s	1025 kg/m ³
Density difference ratio, ϵ	0,025

**Figure 3:** Representation of the steady state analytical solution in the general case

The exact mathematical statement for an interface problem is presented in Figure 2, two regions such that the fresh water and the saline water zones, above and below the interface, respectively. It is shown there Freshwater/saline water with the use of the theoretically established Ghyben-Herzberg principle states that the hydraulic head is more important in the case where the aquifer is shallower. The analytical model also reveals that the penetration length of the saline intrusion depends on the permeability of the aquifer and bedrock. The analytical solution thus implemented (Figure 2) will be used later to model the saline intrusion into the Mboro zone. We apply the analytical solution previously implemented, in the Mboro zone whose depth varies between 2 and 20 m. This analytical model was inspired by the model developed by Albitar [14] and the simulation input data are presented in Table No. 2.

Table 2: Parameters of simulations the zone of Mboro

Parameters	Values
Rate of flow pumping, Q	0,79 m ³ /h
Hydraulic conductivity, K	5,78.10 ⁻⁵ m/s
Aquifer length, L_x	360 m
Annual recharge, I	5 mm/an
Aquifer thickness, h_0	80 m
Density of fresh water, ρ	1000 kg/m
Density of saline water, ρ_s	1025 kg/m ³
Density difference ratio, ϵ	0,025



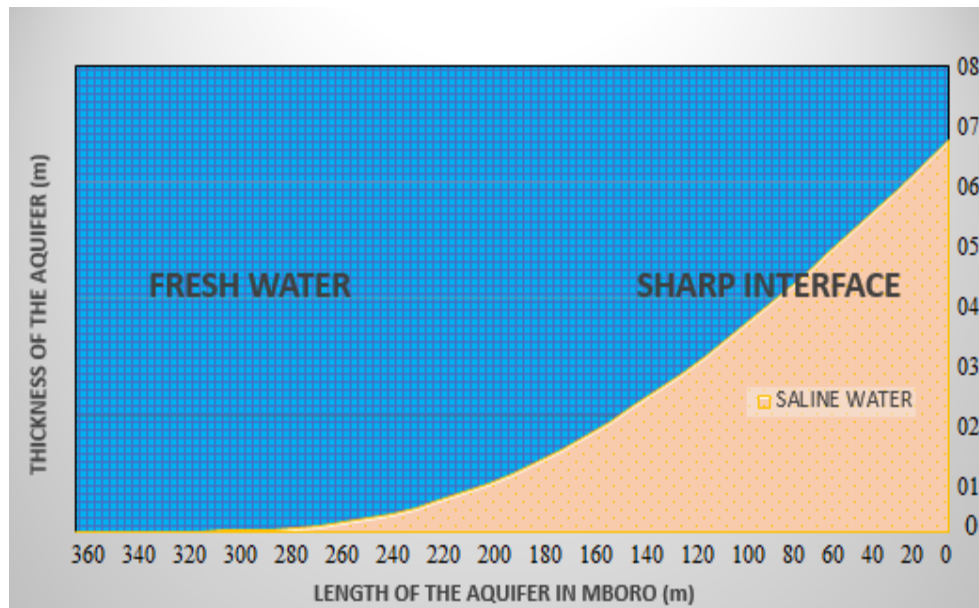


Figure 4: Steady state analytical solution in the Mboro zone

Figure 3 shows the steady state intrusion of seawater into the Mboro zone without a pumping well. In this application, we distinguish a complexity of modeling related to geometry and the nature of the aquifer studied which is characterized by permeability, thickness, well position if there exist. The solution observed here obeys the theory that has been established as a sharp interface approach model above of Figure 2. The penetration of the salt wedge is 280 m well there is a clear interface separating fresh water and saline water whose position depends on the depth of the water table, recharge, water flow as the reality reveals. It can now be confirmed that the analytical solution represents the best behavior of the aquifer if all data are available. However, due to the mathematical complexity of the problem as stated in this way, no analytical solution is available (except for a very small number of specific cases) and various authors have employed different forms of approximations. In spite of the simply achieved by introducing these approximations, the problem still requires a numerical solution for problems of practical interest.

2.4. Numerical modeling

The numerical approach by hydrodynamic modeling constitutes a method in itself to highlight the phenomena of seawater intrusion. Hydrodynamic modeling tools can be used to design localization models of seawater intrusion and to simulate its reaction depending to different scenarios of exploitation, protection or climate change.

In this section of work we use the computational code MARTHE allowing to simulate flow, transport and energy in three dimensional porous medium with finite volume approach [10], [11]. We focus here on the impact of the current pumping, which is studied according to two parameters: the amount of pumping and the position of the wells. The general Darcy equation coupled with the solute mass balance equation referred to as the transport equation is [7, 12, 13]:

$$\left\{ \begin{array}{ll} V = -\frac{K}{\rho g} \nabla P + \rho g \nabla z & \text{in } \Omega \\ h(t=0) = 8 \text{ m} & \text{in } \Omega \setminus \Gamma_o \\ h(t=0) = 9 \text{ m} & \text{on } \Gamma_o \\ \frac{\partial}{\partial t} (\phi C) = \nabla \cdot (\phi D \nabla C) - \nabla \cdot (C V) + Q & \text{in } \Omega \text{ and } t > 0 \\ C = 0 \text{ mg/L} & \text{on } \Gamma_o \text{ and } t > 0 \\ C = 1 \text{ mg/L} & \text{on } \Gamma_e \text{ and } t > 0 \end{array} \right. \quad (13)$$

Where Ω is the interior of the domain; Γ_e et Γ_o are respectively the boundary site East and boundary site West of the domain; $C = C(x, y, z, t)$ is concentration of solute (mg.L^{-1}); ϕ is porosity (m^3/m^3), g gravitation acceleration (m.s^{-2}); P is pressure of fluid (Pa); z is altitude (m); V is velocity of Darcy (m.s^{-1}); D is tensor of



dispersion ($\text{m}^2.\text{s}^{-1}$) and Q is the mass flux injected (source term) (Kg/s.m^3). To have a unique solution, the initial conditions and the boundary conditions are well specified in the model above equation (13). Our aquifer system will be modeled in vertical section. In reality, we considered the Mboro zone with an average depth of between 2 and 20 m. The sea is located on the right boundary coinciding with the eastern boundary (Γ_e), the continental face is located on the left at the western boundary (Γ_o).

a. Presentation of the numerical model MARTHE

MARTHE (Modélisation d'Aquifères par maillage Rectangulaire en régime Transitoire pour le calcul Hydrodynamique des Ecoulements) allows the computation of fluid flows and transfers of mass and energy in three-dimensional porous media with a finite volume approach. The schemes can be simple or complex (unsaturated zone, multiphase flows, vegetation considerations, interaction with rivers,). As part of this project, the MARTHE computational code makes carry out an imposed potential of salt water, which has been set at 0 at the western limit of the model. In the other hand, the concentration of salt is 0 at this limit. The code then calculates a head of fresh water, which evolves linearly within the water column according to the ratio. In the continental boundary, imposed flow conditions have been assigned to the model. This condition significantly influences the shape of the seawater intrusion. To simulate this problem, the equations are discretized in space and time, the MARTHE model is based on a finite volume scheme, with parallelepipedic meshes, which thus comes down to a finite difference scheme [15]. The finite volume method is constructed from an integral formulation based directly on the strong form of the equations to be solved.

Firstly, it is assumed that a first estimate of the concentration, and therefore of the density of the fluid is known at each point and the resulting charges are calculated. From the head gradients, the flow velocities are determined. From the flow velocities, the dispersion convection equation is solved for a better estimate of the concentrations on each point of the domain and this process is repeated for all time steps.

b. Presentation of the numerical model

The geometry that was developed during this study is a simplified conceptual object of the Mboro zone. Indeed, this facilitates and makes more easier the understanding of intrusion phenomena. The geometry of the model has a length of 124 m and a depth of 20 m (Figure 5). The model contains exactly 620 meshes and is discretised sufficiently to be able to observe precisely the actual distribution of the concentrations.

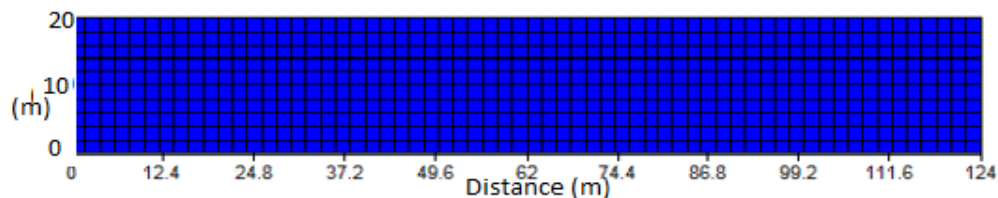


Figure 5: Construction of a control volume mesh from MARTHE

3. Results and Discussions

The parameters assigned to this model for the simulation are the hydrodynamic and hydrodispersive parameters. The hydrodynamic parameters make a very important form of seawater intrusion and we can enumerate the hydraulic conductivity, the Transmissivity and the storage of which their numerical values are presented in the table n°3. The hydrodispersive parameters govern the phenomena relating to the mechanical dispersion of solutes in a porous medium. We can mention the molecular diffusion whose numerical value remains for the whole phase of the simulation at $D_m = 6,6.10^{-6} \text{ m}^2.\text{s}^{-1}$ and the dispersivity α (m) which strongly influences the appearance of the plume of solutes.

Table 3: Input data for the numerical model

Parameters	Values
Storage, S (m^{-1})	1.10^{-4}
molecular diffusion, D_m (m^2/s)	$6,6.10^{-6} \text{ m}^2/\text{s}$
Transmissivity, T (m^2/s)	5.10^{-4}
Porosity, ϕ	0,49



Longitudinal Dispersivity, α_L (m)	10
Transversal Dispersivity, α_T (m)	3
Rate pumping flow, q (m ³ /h)	50

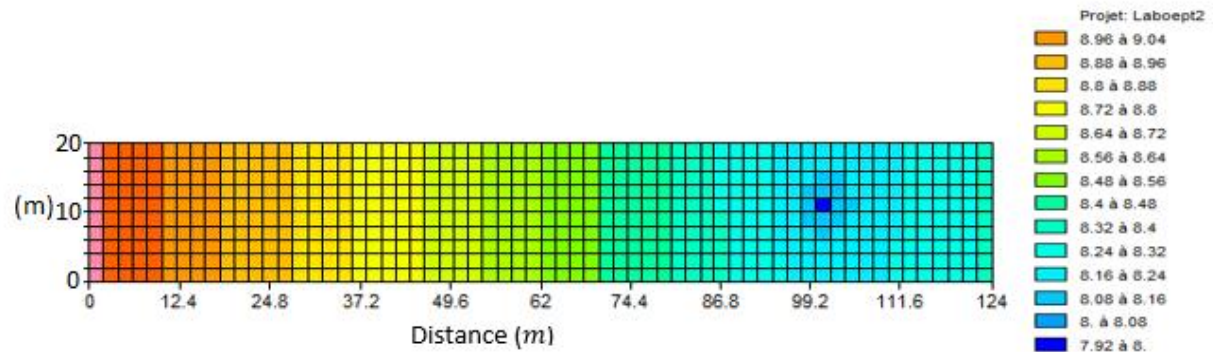


Figure 6: Head distribution with one pumping well

Hydraulic head distribution is shown in Figure 6 at five years of pumping. The location of the transition zone allows us to visualize the hydrodynamic state of the system. Because of high transmissivity, the pumping generates a large drawdown and there is a drop in the hydraulic head only around the well. The latter is more visible in Figure 7 below representing the isovalues of the hydraulic head.

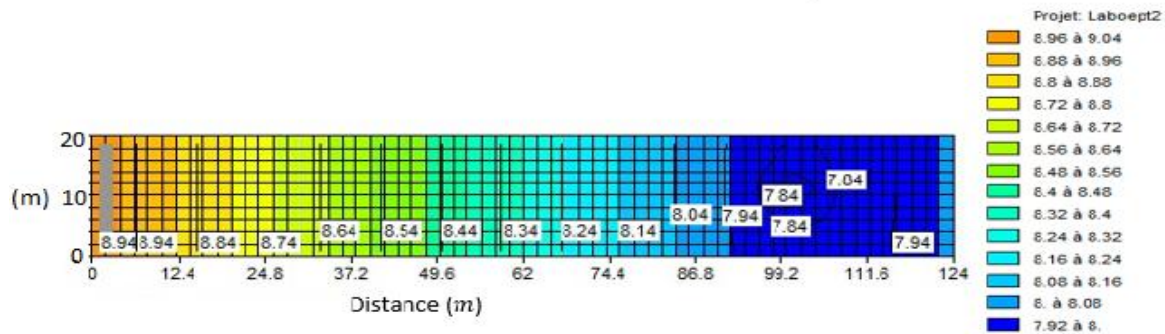


Figure 7: Isovalues of hydraulic head

To estimate the salt concentration, we made simulation scenarios by placing the wells progressively at different positions and varying the pumping rate for three long periods. The Table n°4 summarizes the scenarios and salinity data for the area actually studied.

Table 4: Parameters of simulations scenario

Zone	X (coord)	Y (coord)	Type	Transmissivity (m ² .s ⁻¹)	Chloride Cl (kg.m ⁻³)	Scénario		
Mboro	-16,8916	16,07778	Drilling	5.10 ⁻⁴	37	T5	T10	T60

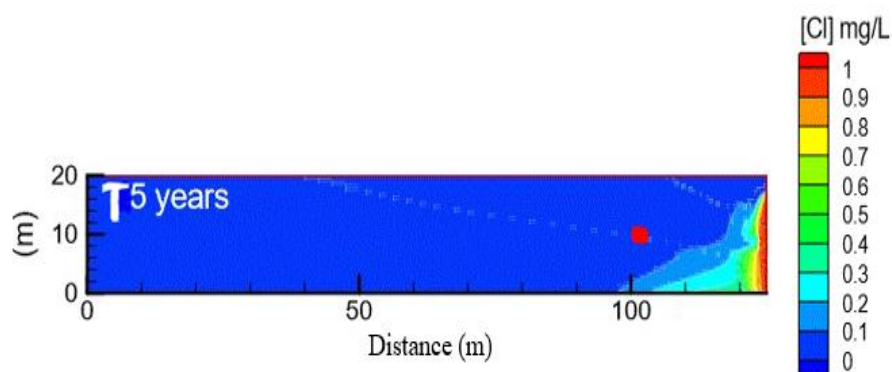


Figure 8: Salt concentration transient at t=10 years



Figure 8 shows a simulation of the transport equation in the presence of a pumping well placed at the coordinate points ($x = 100\text{m}$, $y = 10$) with a flow rate equal to $50\text{m}^3/\text{h}$. The simulation time is more longer and we see that here more than the well is close to the ocean more it becomes contaminate.

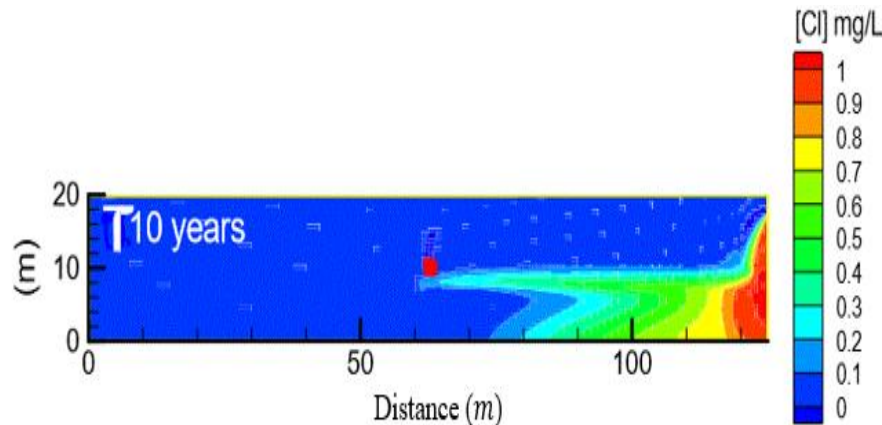


Figure 9: Salt concentration transient at $t=10$ years

Figure 9 shows that even if a well is placed further from the sea inside the aquifer, it may be affected if the pumping is excessive and for a long time. Indeed, we have placed a drilling or well at 62.5 m from the continental boundary very far from the sea but the model shows that at ten years of use the well is partially contaminated by salt by the effect of the diffusion and dispersion.

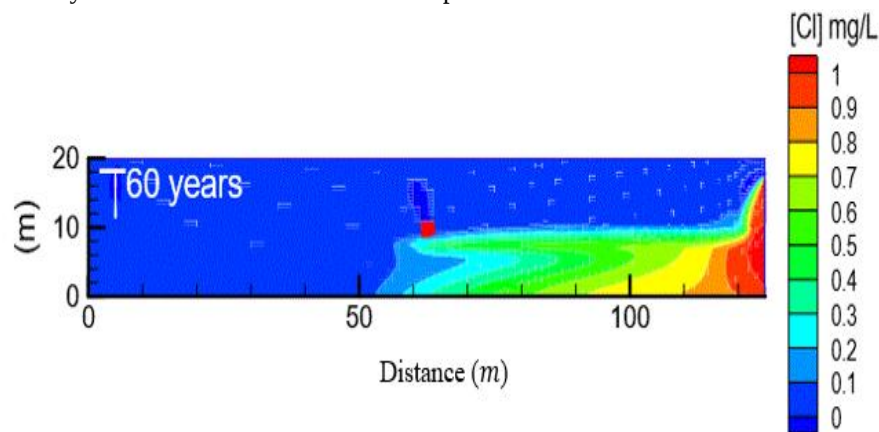


Figure 10: Prevision of the salt concentration for transient state at $t=60$ years

In this figure above, the salinity peak being very close to the structure, this case illustrates a critical situation because the drilling is at the limit of contamination. Indeed, a long continuous exploitation and an increase of the rate flow pumping will lead to salinisation in the future.

4. Conclusion

The saline intrusion represents a major pollution risk for the quaternary sand aquifer of Senegal's northern coast, which is a strategic reservoir for the country. This risk is actually accentuated because of the increase in the frequency and intensity of drought periods, but also in response to the significant population increase in the coastal zone. It will become even more important during the rise of the sea level related to climate change. The good management of this aquifer, therefore, requires better knowledge and better detection of saline intrusions. This work presented a new methodology (modelling) to control saline intrusion into the coastal aquifer of the northern coastline towards Mboro. Two approaches were developed in this study; the first approach was to develop a steady-state analytical model to determine the position of the interface between fresh and salt water in the groundwater. The second was to develop a numerical model to predict the transport of solutes through the porous medium of our study area.

The simulation series take into account the effects of pumping and the positions of the wells in the groundwater. The results obtained by simulating with scenarios suggest that the numerical model thus constructed is



acceptable. The model is able to predict, with good accuracy, the movement and extension of salt water in the northern coastal aquifer.

Thus, this work has allowed us to better understand the dynamics of salt intrusion in the study area but also to have a very important decision-making tool for the rational and sustainable management of groundwater in the Niayes area.

References

- [1]. Mahécor Diouf, Pape Babacar Diop Thioune, Alassane Ba et El Hadji Bamba Diaw, (2017). Impact anthropique et climatique sur la dynamique de la nappe des sables quaternaires du littoral nord du Sénégal, Revue du CAMES –Sciences Appliquées et de l'Ingénieur vol.2 ISSN 2312-8712.
- [2]. DIAW E. B., Mamadou L. Lo, Mamadou Wade, Seydou Ndao, Ablaye Diallo, and Grégoire SISSOKO, (2016). Numerical modeling of solute transport: prediction of the risks of contamination of the Niayes aquifer (Senegal) by pesticides; International Journal of Innovation and Applied Studies, Vol. 17 No.4: 1358-1372.
- [3]. Herzberg A. (1901). Die Wasserversorgung einiger Nordseebäder (The water supply of parts of the North Sea Coast in Germany) ; J. Gasbeleuch. Wasserversorg, 44, 815-819.
- [4]. Peter S. Huyakorn, Peter F. Anderson, James W. Mercer and Harold O. White, JR, (1987). Saltwater Intrusion in Aquifers : Development and Testing of Three-Dimensional Finite Element Model.
- [5]. MALOU Raymond, (2004). Impact of climate on groundwater resources in the Sudano-Sahelian zone. PhD thesis in Natural Sciences of Applied Geology. UCAD, 143p.
- [6]. P.S. Huyakorn, Y.S. Wu and N.S. Park (1996). Multiphase approach to the numerical solution of sharp interface saltwater intrusion problem; Water Resources Research Vol. 32, No.1, pp. 93-102.
- [7]. Bear J, (2010). Modeling Groundwater Flow and Contaminant Transport.
- [8]. Badou-Ghyben, W, (1888). Nota in Verband met de Voorgenomen putboring nabij Amsterdam (Notes on the potable results of well drilling near Amsterdam). In Tijdschrift Van het Kononklijk Institute Van Ingenieurs; The Hague: Amsterdam, Netherlands; Volume 9, pp. 8-22.
- [9]. Omar Kaloum, Ahmet Al Bitar, Jean-P.G. Etchegorry and Mustapha Jazar, (2016). Impact of Demographic on Seawater Intrusion: Case of the Tripoli Aquifer, Lebandon; Water, March 2016.
- [10]. Gionnouloupoulos, P., Mantoglou, A., & Papantoniou, M. (2004). Management of coastal aquifers based on nonlinear optimization and evolutionary algorithms. Journal of Hydrology, 297(1), 209-228.
- [11]. Dominique Thiéry, (2014). Didacticiel du code de calcul Marthe V7.4 ; Exploration des principales fonctionnalités de modélisation des hydrosystèmes. Rapport final BRGM/RP-62798-FR Avril 2014.
- [12]. Jazar, M.; Monneau, M., (2014). Derivation of seawater intrusion model by formal asymptotic. SIAM J. Appl. Math. 74, 1152-1173.
- [13]. Pool, M. & Carrera, J. (2011). A correction factor to account for mixing in Ghyben-Herzberg and critical pumping rate approximations of seawater intrusion in coastal aquifers. Water Ressources Research, 45(5).
- [14]. Albitar, A. (2007). Modélisation des écoulements en milieu poreux hétérogène 2D/3D, avec couplages surface/souterrain et densitaires, Thèse de Doctorat de de l'Institut National Polytechnique de Toulouse, 151p.
- [15]. Dominique Thiéry, (2014). Modélisation 3D des écoulements à densité variable avec le logiciel MARTHE version 6.9. Rapport final BRGM/RP-55871-FR, Octobre 2007, révisé Mai 2014.

